3 (Sem–6/CBCS) PHY HE 5

2023

PHYSICS

(Honours Elective)

Paper : PHY–HE–6056

(Classical Dynamics)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×10=10

   (a) What is conservative force ? Give examples.

   (b) A gas particle is moving inside a sphere, what will be the type of the constraint in it ?

   (c) What will be the number of generalized coordinates of a system of N particles having k constraints of motion ?

(d) Calculate the potential energy function associated with the force

$\vec{F} = -yz\hat{i} - zx\hat{j} - xy\hat{k}$.

(e) Write down the equation of stable equilibrium of system executing small oscillations.

(f) Under what condition Galilean transformation reduces to Lorentz transformation ?

(g) Justify that the earth is not an inertial frame.

(h) State the basic postulates of relativity.

(i) What is critical velocity of a fluid ? Write down its expression in terms of Reynold's number.

(j) Define terminal velocity of an object falling through a fluid.

2. Answer the following questions : 2×5=10

(a) What are generalized coordinates ? If a generalized coordinate has the dimension of momentum, what would be the dimension of generalized velocity ?

(b) Write the Lagrangian of a charged particle in an electromagnetic field, explaining each term.

(c) Explain : *(i)* normal modes of vibration; *(ii)* normal coordinates and *(iii)* normal frequencies of a system which undergoes small oscillations.

(d) 1 *kg* of a substance is fully converted into energy. Calculate the amount of energy produced.

(e) Show that the velocity of flow is inversely proportional to the area of cross-section of the tube.

3. Answer the following : ***(any four)*** 5×4=20

(a) Show that the path of a charged particle moving perpendicular to an uniform electric field is parabolic in nature.

(b) Write down the Lorentz transformation equations and obtain Lorentz-Fitzgerald contraction formula.

(c) A massless spring of force constant k has masses m_1 and m_2 attached to its two ends. The system rests on frictionless horizontal plane. Show that the angular frequency of the system is $\sqrt{\frac{k}{\mu}}$ where $\mu = \frac{m_1 m_2}{m_1 + m_2}$.

(d) What is relativistic energy? Show that the relativistic energy E of a particle is given by is $E^2 = p^2c^2 + m^2c^4$.

(e) Explain relativistic time dilation. A certain particle has a life time of 10^{-6} *sec* when measured at rest. How far does it go before decaying if its speed is $0.9c$ when it is created (c is the velocity of light in free space)?

(f) Obtain the equation of continuity for an incompressible fluid. An incompressible fluid is flowing through a pipe. If the diameter of the pipe is doubled, what change will occur to the velocity of the fluid?

4. Answer the following : ***(any four)*** 10×4=40

(a) Obtain Lagrange's equation of motion from D'Alembert's principle. Consider a particle of mass m moving in a plane in a central force field. Write its Lagrangian in plane polar co-ordinates. Write the equations of motion and obtain the differential equation of the orbit. 5+5=10

(b) Derive Hamilton's canonical equation of motion from Hamilton's principle of least action. A mass m is suspended by a massless spring of spring constant k. The suspension point is pulled upwards with constant acceleration a_0. Find the Hamiltonian of the system and Hamilton's equations of motion. Also find the equation of motion of the system. 5+5=10

(c) Deduce Newton's second law of motion from Hamilton's principle. Masses m and $2m$ are connected by a light inextensible string which passes over a pulley of radius a. Write the Lagrangian and find the acceleration of the system. 5+5=10

(d) Derive the velocity addition theorem using four vectors. A particle of rest mass $1.67 \times 10^{-27} kg$ moves with a velocity $c/\sqrt{2}$. Calculate its relativistic momentum, kinetic energy and total energy. 6+4=10

(e) Establish Einstein's mass-energy equivalence principle $E = mc^2$. A pion decays into a muon and a neutrino. Show that momentum of muon is given by $p_\mu = \frac{c\left(m_\mu^2 - m_\pi^2\right)}{2m_\pi}$. 6+4=10

(f) State and explain relativistic transverse Doppler effect. Explain blue shift and red shift in Doppler effect. Calculate the wavelength shift in the relativistic Doppler effect for the frequencies H_α line ($6563 \AA$) emitted by a star receding from the earth with a relative velocity $0.1c$. Is the classical result (first order) a good approximation? 2+3+5=10

(g) Derive Poiseuille's equation for the rate of flow of fluid through a capillary tube. A tube of $0.1\,cm$ in diameter and $40\,cm$ long has to deliver $20\,kg$ water in $25\,min$. Calculate the constant pressure head to be maintained. The coefficient of viscosity for water is $8.9 \times 10^{-4} N\text{-}s\text{-}m^{-2}$. 6+4=10

(h) What is Reynold's number? Mention various range of values of Reynold's number for different types of flow of fluid. Derive the Navier-Stokes equation for flow of a viscous fluid. 1+3+6=10
