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**3 (Sem-4/CBCS) PHY HC 1**

**2023**

**PHYSICS**

(Honours Core)

Paper : PHY-HC-4016

**(Mathematical Physics-III)**

Full Marks : 60

Time : Three hours

**The figures in the margin indicate full marks for the questions.**

1. Give short answers to the following questions : 1×7=7

(a) Find the principal value of  $i^i$ .

(b) Define a multiply connected region in complex plane.

(c) Find the value of  $L^{-1}\left\{\frac{1}{s(s-a)}\right\}$  for

$s > a$ .

Contd.

(d) What does the equation  $|z - i| = 2$  represent ?

(e) State convolution theorem of Fourier transform.

(f) Write the transformation rule for a covariant tensor of rank two.

(g) Plot the number  $e^{(1+i\pi/4)}$ .

2. Answer the following questions:  $2 \times 4 = 8$

(a) Define simple pole and essential singularity.

(b) Establish the shifting property of Fourier transform.

(c) Find inverse Laplace transform of  $t^{-\frac{1}{2}}$ .

(d) Show that the number of independent components of a skew-symmetric tensor of rank 2 in  $n$ -dimensional space is

$$\frac{n(n-1)}{2}.$$

3. Answer **any three** questions of the following:  $5 \times 3 = 15$

(a) Check the analyticity and hence find derivative of the function  $f(z) = \sin z$ .

$$3 + 2 = 5$$

(b) Find the value of the integral

$$\int_0^{1+i} (x - y - ix^2) dz \text{ along real axis from}$$

$z = 0$  to  $z = 1$  and then along the line parallel to imaginary axis from  $z = 1$  to  $z = 1+i$ .

- (c) Find the Fourier sine transform of a function defined by

$$f(t) = \begin{cases} \cos \alpha t, & 0 \leq t \leq \frac{\pi}{\alpha} \\ 0, & t > \frac{\pi}{\alpha} \end{cases}$$

- (d) Evaluate :

$$L^{-1} \left\{ \frac{(s+1)}{s^2(s+2)^3} \right\}$$

- (e) Define Levi-Civita symbol in three dimensional space. Show that

$$(\vec{A} \times \vec{B})_i = \epsilon_{ijk} A_j B_k \quad 2+3=5$$

4. Answer the following questions : **(any three)**

10×3=30

- (a) (i) Prove Cauchy-Riemann conditions for analytical functions. What is the sufficient condition for a function to be analytic ? 4+1=5

- (ii) Show that  $|z_1 + z_2| \leq |z_1| + |z_2|$

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- (iii) Give Laurent series expansion for function  $f(z)$ .

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- (b) (i) What are symmetric and antisymmetric tensors ? Show that every tensor can be expressed as the sum of two tensors, one of which is symmetric and the other antisymmetric in a pair of co-variant or contravariant indices.

2+3=5

- (ii) What is Kronecker delta ? Prove that Kronecker delta is a mixed tensor of second rank.

1+4=5

- (c) (i) Define Laplace transform of a function  $F(t)$ . Show that

$$L(1) = \frac{1}{s}, \quad s > 0 \text{ and}$$

$$L(e^{kt}) = \frac{1}{s-k}, \quad s > k \quad 1+2+2=5$$

(ii) Find the inverse Laplace transform of

$$\frac{6}{2s-3} - \frac{3+4s}{9s^2-16} + \frac{8-6s}{16s^2+9} \quad 5$$

(d) Find the Fourier transform of

$$f(x) = \begin{cases} 1-x^2, & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$$

Hence, evaluate :

$$\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} \cos \frac{x}{2} dx$$

(e) Evaluate *any two* of the following integrals : 5×2=10

(i)  $\int_{-\infty}^{+\infty} \frac{\sin x}{x} dx$

(ii)  $\int_0^{\infty} \frac{dx}{x^2+1}$

(iii)  $\int_0^{2\pi} \frac{d\theta}{5+4\cos\theta}$

(f) (i) The Laplace transform of  $\sin 3t = \frac{3}{s^2+9}$  and the Laplace

transform of  $\cos 5t = \frac{s}{s^2+25}$ .

Find the Laplace transform of  $5 \sin 3t + 3 \cos 5t$  using linearity property of Laplace transform. 5

(ii) Find the inverse Laplace transform of  $4s+5/(s-1)^2(s-2)$ . 5