2019

PHYSICS

(Honours)

Paper : PHY–HC–1026

(Mechanics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer (any *seven*) : 1×7=7

   (a) An observer is moving with velocity of light c along x-axis. A light beam moves parallel to the observer. If you believe in Galilean transformation, what will be the velocity of the light beam according to the observer?

   (i) 0

   (ii) $2c$

   (iii) c

   (iv) $-2c$

(b) 150 J amount of work has been done in moving a mass of 150 kg from the state of rest. The final velocity of the mass is

(i) 150 m/sec

(ii) 2 m/sec

(iii) $\sqrt{2}$ m/sec

(iv) 4 m/sec

(c) What will be the distance of the centre of mass of the Earth-Moon system from the centre of the Earth?

Given the distance between Earth and Moon as $d = 3 \cdot 82 \times 10^8$ m, the mass of the Earth as $m_1 = 5 \cdot 98 \times 10^{24}$ kg and the mass of the Moon as $m_2 = 7 \cdot 36 \times 10^{22}$ kg.

(i) $9 \cdot 28 \times 10^6$ m

(ii) $4 \cdot 64 \times 10^6$ m

(iii) $3 \cdot 77 \times 10^8$ m

(iv) $1 \cdot 88 \times 10^8$ m

(Continued)

(d) The potential energy of a conservative force is expressed as $U = \frac{1}{4}x^4$. The force at $x = 2$ is

(i) $F = 4$

(ii) $F = -4$

(iii) $F = 8$

(iv) $F = -8$

(e) A pendulum oscillates with such a high energy that the swinging angle θ goes up to $\theta = 180°$. At this position, which of the following statements is correct?

(i) The energy of the pendulum goes to zero

(ii) The pendulum is in stable equilibrium

(iii) The pendulum is in unstable equilibrium

(iv) The pendulum stays indefinitely at that position

(Turn Over)

(f) Newton's second law for linear motion is given by $F = ma$. Which of the following quantities is analogous to mass?

(i) Torque

(ii) Moment of inertia

(iii) Angular acceleration

(iv) Angular momentum

(g) A neutrino emitted in a supernova explosion moves with 90% of the speed of light. Its mass increases by a factor

(i) 90

(ii) 1/90

(iii) 2·29

(iv) 1/2·29

(*Continued*)

(h) According to special relativity, the energy output of the stars is due to conversion of mass into energy occurring from nuclear reactions. The rate of energy liberated by the Sun is 10^{26} J/sec. If the mass of the Sun is 2×10^{30} kg and present age of the Sun is 5×10^{9} years, what is the approximate mass that has been radiated out in the form of energy?

(i) $1 \cdot 75 \times 10^{26}$ kg

(ii) $1 \cdot 75 \times 10^{25}$ kg

(iii) 10^{9} kg

(iv) 2×10^{30} kg

(i) The mechanical energy of a block-spring system having spring constant 1·3 N/m and oscillation amplitude 2·4 m is

(i) 3·744 J

(ii) −3·744 J

(iii) 3·12 J

(iv) 1·56 J

(*Turn Over*)

(j) The semimajor axis of a planet is double that of the Earth. The planet is 4 times massive than the Earth. Its period of revolution around the Sun is

(i) 4 years

(ii) 2 years

(iii) $\frac{1}{4}$ year

(iv) 2·82 years

2. Answer any *four* questions : 2×4=8

(a) A force acts on a particle of mass 3 kg along x-axis in such a way that the position of the particle as a function of time is given by

$$x(t) = 3{\cdot}0t - 4{\cdot}0t^2 + 1{\cdot}0t^3$$

Here x is in metre and t is in second. Find the work done on the particle during the time interval $t = 0 - 4{\cdot}0$ sec.

(b) Find the torque at time $t = 1$ sec, if the angular momentum of a particle is expressed as

$$\vec{L} = 6t^2\hat{i} - (2t+1)\hat{j} + (12t^3 - 8t^2)\hat{k}$$

(c) The potential energy of a diatomic molecule is given by

$$V(r) = \frac{a}{r^{12}} - \frac{b}{r^6}$$

Show that the potential energy at equilibrium separation is $V = \frac{-b^2}{4a}$.

(d) The moment of inertia of a rotating star changes to 1/3 of its initial value. What is the ratio of its new rotational energy to its initial rotational energy?

(e) The Martian satellite Phobos orbits the planet in a circular orbit with period 7 h 39 min. The orbital radius is $9{\cdot}4 \times 10^6$ m. Calculate the mass of the Mars from these data.

Or

An asteroid headed directly to the Earth has an initial velocity $V_i = 12$ km/sec relative to the planet when it is at a distance equal to 10 times the radius of the Earth (R_E). Show that the final velocity (V_f) of the asteroid when it reaches very close to the Earth's surface is related to its initial velocity as

$$V_f = \left\{V_i^2 + \frac{2GM}{R_E}\left(1-\left(\frac{1}{10}\right)\right)\right\}^{\frac{1}{2}}$$

(f) What is the meaning of null result of Michelson-Morley experiment? The energy of a relativistic particle is given by $E^2 = m^2c^4 + c^2p^2$, where p is the momentum and m is the rest mass of the particle. If $pc << mc^2$, show that the energy of the particle can be approximated as

$$E = \pm\left(mc^2 + \frac{P^2}{2m}\right)$$

3. Answer any *three* questions : 5×3=15

(a) Construct the equation of a damped oscillator in the following form :

$$\ddot{x}(t) + 2b\dot{x}(t) + \omega^2 x(t) = 0$$

What is the condition of overdamped oscillation? Neatly sketch the graph $x(t)$ for overdamped, critically damped and underdamped oscillations. 2+1+2=5

(b) The square of velocity of a particle moving in a central force $F(r)$ is given by $V^2 = \dot{r}^2 + r^2\dot{\theta}^2$. Show that the total energy of the particle can be expressed as

$$E = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) - \int F(r)\,dr$$

Show that areal velocity $\frac{dA}{dt}$ of a particle moving in central force is half of the angular momentum per unit mass h. For a central force $F(r) = -K/r^2$ $(K > 0)$, show that the radial velocity of the particle is

$$\dot{r} = \sqrt{\frac{2}{m}\left\{\left(E + \frac{K}{r}\right) - \frac{mh^2}{2r^2}\right\}}$$

1+2+2=5

(c) Define elastic and inelastic collisions. An alpha particle of mass 4 u experiences an elastic head-on collision with a gold nucleus of mass 197u that is originally at rest (the symbol u represents atomic mass unit $1 \cdot 67 \times 10^{-27}$ kg). What percentage of its initial kinetic energy does the alpha particle lose? 1+1+3=5

(d) A fluid with coefficient of viscosity η is flowing through a capillary tube of radius a, cross section A and length l. The pressure difference is P. Consider a cylindrical cross section of radius x inside the tube. If $\frac{dv}{dx}$ is the velocity gradient, show that

$$\frac{dv}{dx} = -\left(\frac{P}{2l\eta}\right)x$$

Integrate the above equation with the boundary condition $v = 0$ for $x = a$ and show that the velocity of the fluid at any point x away from the axis of the tube is

$$v = \frac{P}{4l\eta}(a^2 - x^2)$$

2+3=5

(e) A solid sphere of mass $M = 2 \cdot 5$ kg starts rotating at $t = 0$ sec with the constant angular acceleration $\alpha = 24$ rad/sec^2. If the radius of the sphere is $R = 0 \cdot 20$ m, what is its rotational kinetic energy at $t = 2 \cdot 5$ sec? 5

4. Answer any *three* questions : 10×3=30

(a) Deduce Poiseuille's formula $V = \pi P a^4 / 8\eta l$ for rate of flow of a fluid of coefficient of viscosity η flowing through a tube of radius a and length l. A pressure head with height $h = 0 \cdot 20$ m is used to find viscosity of a fluid. If the capillary tube with $0 \cdot 0012$ m diameter and $0 \cdot 26$ m length is used, find the viscosity of the fluid. Given that the volume of fluid collected in 60 second is $8 \cdot 1 \times 10^{-7}$ m^3. The density of the fluid is 900 kg/m^3 and acceleration due to gravity is 9·8 m/sec^2. 7+3=10

(b) Deduce the relation between Young's modulus Y, bulk modulus K and rigidity modulus η in the following form :

$$\frac{9}{Y} = \frac{1}{K} + \frac{3}{\eta}$$

What twisting torque must be applied to a 1 m long wire with 1 mm diameter, in order to twist one end of it through 90°, the other end remaining fixed? Given the rigidity modulus is 2×10^{10} N/m^2. 7+3=10

(c) What are the two postulates of special relativity? The Lorentz transformation equation for time is given by

$$t' = \gamma(t - vx/c^2)$$

Show that if two events are simultaneous in one frame, they are not simultaneous in other frame moving relative to the former with velocity v. The galaxy NGC 7319 is at a distance of 3×10^8 light-years. The oxygen line of rest wavelength 513 nm is observed to be Doppler shifted to 525 nm. What is the speed of the galaxy as observed from the Earth? A cosmic ray proton is moving with 98% of the speed of light. Calculate the kinetic energy of the proton. Given mass of the proton as $m = 1{\cdot}67 \times 10^{-27}$ kg. Express your answer in MeV. 2+2+3+3=10

(d) The displacement of a particle undergoing damped harmonic oscillation is given as

$$x(t) = x_{max} e^{-bt/2m} \cos\left(\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}\right)t$$

Here the symbols have their usual meanings. If mechanical energy of the oscillator is $\frac{1}{2}$ of the quantity kA^2, where A is the amplitude of oscillation, show that energy of the damped oscillator decreases exponentially. What is the condition of under-damped oscillation? An oscillator has the following quantities :

$m = 0{\cdot}250$ kg, $k = 85$ N/m, $b = 0{\cdot}07$ kg/sec

Show that it is an underdamped oscillator. Calculate the period of the oscillation. How long does it take for the mechanical energy to drop to 1/4th of its initial value? 2+1+2+2+3=10

(e) If x_i and x_f are initial and final positions of a block attached to a spring of spring constant k, show that

$dx'_1 = \frac{x_1 - vt}{\sqrt{1 - v^2/c^2}}$ $t' = \frac{t - vx/c}{\sqrt{1 - v^2}}$

work done by the spring force is $W = (1/2)k(x_i^2 - x_f^2)$. The differential displacement in XY plane is given by the vector $\vec{dr} = dx\hat{i} + dy\hat{j}$. What is the work done by a force $\vec{F} = (3x^2 \text{N})\hat{i} + (4\text{N})\hat{j}$ (x is in meter) in displacing the particle from the point (2 m, 3 m) to the point (3 m, 0 m)? An wooden block of mass 14 kg is pushed by a person with horizontal force of magnitude 40 N. The displacement of the block on the floor is 0·50 m. The speed of the block decreases from $V_0 = 0{\cdot}60$ m/sec to $V = 0{\cdot}20$ m/sec due to friction on the floor. What is the increase in thermal energy of the floor due to friction between the block and the floor? With a neat potential energy diagram, show the stable equilibrium and unstable equilibrium.

2+3+3+2=10

(f) Two particles of masses m_1 and m_2 undergo elastic collision. If the second particle (m_2) is initially at rest and if

(*Continued*)

V_{1i} is the initial velocity of the first particle (m_1), show that the final velocities of the two particles are given by

$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i}$$

$$V_{2f} = \frac{2m_1}{m_1 + m_2} V_{1i}$$

Discuss the cases $m_1 = m_2$, $m_2 >> m_1$ (massive target) and $m_1 >> m_2$ (massive projectile). If both the bodies are moving before collision, then prove the following relation

$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i} + \frac{2m_2}{m_1 + m_2} V_{2i}$$

When spacecraft Voyager 2 approached Jupiter with velocity of 12 km/sec, Jupiter was moving with a velocity of 13 km/sec. The spacecraft then turns back in opposite direction. By making reasonable assumption about the masses, estimate the final velocity of Voyager 2. 4+1+1+1+3=10

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