2019

MATHEMATICS

(Honours)

Paper : MAT–HC–1016

(Calculus)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) Define hyperbolic sine and cosine functions.

(b) When is a point on a curve said to be a point of inflection?

(c) Can L'Hopital's rule be applied to evaluate limits which are not in indeterminate form?

(d) How is a surface of revolution generated?

(e) Write down the domain of the vector function

$$\vec{F}(t) = 2t\hat{i} - 3t\hat{j} + \frac{1}{t}\hat{k}$$

(f) Evaluate :

$$\int_0^1 (t\hat{i} + e^{2t}\hat{j} + 3\hat{k})\,dt$$

(g) State Kepler's second law of motion.

2. Answer the following questions : 2×4=8

(a) Given $f(x) = x^3 - 3x^2 + 1$. Use second derivative of f to determine the intervals on which f is concave up and concave down. Is there any inflection point on the curve?

(b) Find the arc length of the curve $y = x^{3/2}$ on the interval [0, 5].

(c) Parameterize the curve $r = 2\cos^3\theta$.

(d) Find the volume of the parallelopiped determined by the vectors $\vec{u} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{v} = 5\hat{i} + 9\hat{j} - \hat{k}$ and $\vec{w} = -4\hat{i} + 7\hat{j} - 11\hat{k}$.

3. Answer any *three* of the following questions :

(a) Sketch the graph of the following equations (any *one*) :

(i) $y = \dfrac{3x-5}{x-2}$

(ii) $y = \dfrac{2x^2-8}{x^2-16}$

identifying the locations of intercepts, inflection points (if any) and asymptotes. 5

(b) State Leibnitz's rule on the *n*th derivative of product of two functions. Use this rule to find *n*th derivative of $e^{ax}\cos bx$. 1+4=5

(c) Obtain the reduction formula for

$$\int \sin^n x\, dx$$

Hence evaluate $\int \sin^4 x\, dx$. 3+2=5

(d) Find the volume of the solid generated when the region bounded by the parabola $y = x^2$ and the line $y = x$ revolved about *y*-axis (use any method). 5

(e) Find the tangent vector to the graph of the vector function $\vec{F}(t)$ at the point P_0 corresponding to $t_0 = -1$, where

$$\vec{F}(t) = (t^{-3}, t^{-2}, t^{-1})$$

Also find the parametric equations of the tangent line at $t_0 = -1$. 2+3=5

Answer any *three* of the following questions :

4. *(a)* Evaluate the following using L'Hopital's rule : 3+2=5

(i) $\underset{x \to \infty}{\text{Lt}} \left(1 + \frac{1}{x}\right)^x$

(ii) $\underset{x \to 0}{\text{Lt}} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$

(b) A travel company plans to sponsor a special tour to Africa. There will be accommodations for no more than 40 people, and the tour will cancelled if no more than 10 people book reservations. Based on past experience, the manager determines that if n people book the tour, the profit (in ₹) may be modelled by the function

$$P(n) = -n^3 + 27 \cdot 6n^2 + 970 \cdot 2n - 4235$$

For what size tour group is profit maximized? 5

5. *(a)* Find the area of the surface generated by revolving about the x-axis, the top half of the cardioid $r = a(1+\cos\theta)$ for $0 \le \theta \le \pi$. 5

(b) Using cylindrical shell method, find the volume of the solid formed by revolving the region R bounded by the curve $y = x^{-2}$ and the x-axis for $1 \le x \le 2$ about the line $x = -1$. 5

6. *(a)* If $\vec{F}(t) = \hat{i} + e^{t}\hat{j} + t^{2}\hat{k}$ and $\vec{G}(t) = 3t^{2}\hat{i} + e^{-t}\hat{j} - 2t\hat{k}$, find

$$\frac{d}{dt}[\vec{F}(t) \cdot \vec{G}(t)]$$ 3

(b) The position vector for a particle in space at time t is given by

$$\vec{R}(t) = \cos t\hat{i} + \sin t\hat{j} + 3t\hat{k}$$

Find the velocity vector and the direction of motion at time $t = \pi/4$. 3

(c) Find the tangential and normal components of acceleration of an object that moves with position vector

$$\vec{R}(t) = t\hat{i} + t^{2}\hat{j}$$ 4

7. A boy standing at the edge of a cliff throws a ball upward at an angle of 30° with an initial speed 64 ft/s. Suppose that when the ball leaves the boy's hand, it is 48 ft above the ground at the base of the cliff :

(a) What are the time of flight of the ball and its range?

(b) What are the velocity of the ball and its speed at impact?

(c) What is the highest point reached by the ball during its flight? 4+3+3=10

8. (a) Evaluate : 2

$$\underset{x \to +\infty}{\mathrm{Lt}} \frac{2x^2 - 3x + 1}{3x^2 + 5x - 2}$$

(b) Determine whether the graph of the function

$$f(x) = x^{2/3}(2x + 5)$$

has a vertical tangent or a cusp. 4

(c) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y-axis. 4

Or

Obtain the reduction formula for

$$\int (\log x)^n \, dx$$

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a) $\frac{2v_0 \sin\theta}{g}$

b) $\frac{v_0^2 \sin 2\theta}{g}$

c) $\frac{(v_0 \sin\theta)^2}{g}$

$D^n(uv) = D^n(u) + {}^nC_1 D^{n-1}(u) D(v) - {}^nC_2 D^{n-2}(u) D^2(v) \ldots\ldots + {}^nC_n D^n(v)$