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3 (Sem-1/CBCS) MAT HC 1

2020

(Held in 2021)

MATHEMATICS

(Honours)

Paper : MAT-HC-1016

(Calculus)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 7 = 7$

(a) Write down the value of

$$\lim_{x \rightarrow +\infty} e^{-x} \cos x.$$

(b) When the line $x=c$ is a vertical asymptote of the graph of a function $f(x)$?

Contd.

- (c) Define scalar triple product of three vectors $\vec{u}, \vec{v}, \vec{w}$.
- (d) When Washer Method is used to compute a volume of revolution ?
- (e) Determine the values of t for which the function $\vec{G}(t) = t\hat{i} - \frac{1}{t}\hat{j} + \frac{1}{t-1}\hat{k}$ is continuous.
- (f) Under what condition the graph of a vector function $\vec{F}(t)$ is smooth ?
- (g) For a production process, $C(x)$ denotes total production cost of x units and $R(x)$ denotes total revenue derived from the sale of that x units. Under what condition profit will be maximum ?

2. Answer the following questions : $2 \times 4 = 8$

- (a) Examine if $f(x) = x^4$ has a point of inflection at $x = 0$.
- (b) Find the n th derivative of $y = xe^{ax}$, using Leibniz's rule.
- (c) Obtain the reduction formula for $\int \tan^n x \, dx$.

(d) Evaluate $\int_0^{\pi} (\hat{t}i + 3\hat{j} - \sin t\hat{k}) dt$.

3. Answer **any three** of the following questions :

(a) If $y = \sin(m \sin^{-1} x)$, show that $2+3=5$

(i) $(1 - x^2)y_2 - xy_1 + m^2y = 0$

(ii) $(1 - x^2)y_{n+2} = (2n + 1)xy_{n+1} + (n^2 - m^2)y_n$

(b) Sketch the graph of the following (**any one**) equation. 5

(i) $y = 4 + \frac{2x}{x-3}$

(ii) $g(t) = (t^3 + t)^2$

identifying the locations of intercepts, inflection points (*if any*) and asymptotes.

(c) Obtain the reduction formula for $\int \sin^m x \cos^n x dx$. 5

(d) Using Washer method find the volume of the solid formed by revolving the region bounded by $x = y^2$ and $y = x^2$ about (i) x -axis, (ii) y -axis. 5

- (e) The position vector of a moving body is $\vec{R}(t) = 2t\hat{i} - t^2\hat{j}$ for $t \geq 0$. Express \vec{R} and velocity vector $\vec{v}(t)$ in terms \hat{u}_r and \hat{u}_θ , \hat{u}_r and \hat{u}_θ being unit vectors along and perpendicular to radial axis. 5

Answer **any three** of the following questions :

4. (a) Evaluate the following using L' Hôpital's rule

$$(i) \quad \lim_{x \rightarrow 0} \frac{\log x}{\operatorname{cosec} x} \quad 2$$

$$(ii) \quad \lim_{x \rightarrow +\infty} \left(1 + \frac{1}{2x} \right)^{3x} \quad 3$$

- (b) A liquid form of antibiotic manufactured by a pharmaceutical firm is sold in bulk at a price of Rs. 200 per unit. If the total production cost (in Rs.) for x units is $C(x) = 5,00,000 + 80x + .003x^2$ and if the production capacity of the firm is atmost 30,000 units in a specified time, how many units of antibiotic must be manufactured and sold in that time to maximize the profit? 5

5. (a) Find the length of the curve defined by $9x^2 = 4y^3$ between the points $(0, 0)$ and $(2\sqrt{3}, 3)$. 3
- (b) Find the length of the polar curve $r = \cos\theta$. 3
- (c) Using cylindrical shell method find the volume of the solid formed by revolving the region bounded by the lines $y = 2x$, the y -axis and $y = 1$ about y -axis. 4
6. (a) Given $\vec{F}(t) = \hat{i} + t\hat{j} + t^2\hat{k}$ and $\vec{G}(t) = t\hat{i} - e^t\hat{j} + 3\hat{k}$. Find $\frac{d}{dt}[\vec{F}(t) \times \vec{G}(t)]$. 3
- (b) Find a vector function \vec{F} whose graph is the curve of intersection of the hemisphere $z = \sqrt{4 - x^2 - y^2}$ and the parabolic cylinder $y = x^2$. 2

- (c) A projectile travels in vacuum in a co-ordinate plane, with x -axis along the level ground. If the projectile is fired from a height of s_0 with initial speed v_0 and angle of elevation α , then prove that at time t ($t \geq 0$) it will be at the point $(x(t), y(t))$ where

$$x(t) = v_0 \cos \alpha \cdot t \text{ and}$$

$$y(t) = v_0 \sin \alpha \cdot t - \frac{1}{2}gt^2 + s_0. \quad 5$$

7. (a) Examine if $f(x) = x^{1/3}(x-4)$ has a vertical tangent at $x=0$. 2

- (b) If a non-zero vector function $\vec{F}(t)$ is differentiable and has constant length, then $\vec{F}(t)$ is orthogonal to the derivative vector $\vec{F}'(t)$ — Prove it. 3

- (c) An object moving along a smooth curve (with $T' \neq 0$) has velocity \vec{V} given by $\vec{V} = \frac{ds}{dt} \hat{T}$.

Deduce the expression for acceleration in the form

$$\vec{A} = \frac{d^2s}{dt^2} \hat{T} + k \left(\frac{ds}{dt} \right)^2 \hat{N}$$

(Symbols having their usual meanings.)

5

8. (a) A manufacturer estimates that when x units of a particular commodity are produced each month, the total cost (in Rs.) will be

$$C(x) = \frac{1}{8}x^2 + 4x + 200$$

and all units can be sold at a price of $p(x) = 49 - x$ rupees per unit.

Find average cost, marginal cost and marginal revenue for this production process. 2+2+2=6

- (b) Derive the formula for surface area of a sphere of radius r . 4
