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3 (Sem – 1) MAT M 1

2021

(Held in 2022)

MATHEMATICS

(Major)

Paper : 1·1

(Algebra and Trigonometry)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 10 = 10$

(a) Give an example of a relation on the set of real numbers R which is reflexive and transitive but not symmetric.

(b) Is generator of a cyclic group always unique ?

(c) Define Hermitian matrix.

(d) Find all partitions of the set $x = \{1, 2, 3\}$.

(e) Find the value of i^i.

(f) Find the rank of the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{pmatrix}$$

(g) Examine whether the inverse of the matrix $\begin{pmatrix} 1 & w \\ w & w^2 \end{pmatrix}$ exists or not.

(h) Define an operation $*$ on the set of real numbers R where
$a * b = a + 2b, \ \forall a, b \in R$

(i) What is normal form of a matrix ?

(j) Find the amplitude of the complex number $-1-i$.

2. Give the answer of the following : $2 \times 5 = 10$

(a) Can a non-Abelian group have an Abelian subgroup ? Justify your answer.

(b) If $f : A \to B$ and $g : B \to C$ are bijective mappings, then prove that $g \circ f$ is also a bijective mapping.

(c) Prove that

$$\pi = 2\sqrt{3}\left(1 - \frac{1}{3 \cdot 3} + \frac{1}{5 \cdot 3^2} - \frac{1}{7 \cdot 3^3} + \cdots\right)$$

(d) Solve the equation $x^3 + 6x + 20 = 0$ if one root is $1 + 3i$.

(e) Show that the relation defined on $N \times N$ by $(a, b) \sim (c, d)$ iff $a + d = b + c$ is an equivalence relation.

3. Answer ***any four*** : 5×4=20

(a) Define an equivalence relation on a nonempty set. Show that the relation 'congruence modulo m' is an equivalence relation on the set of integers. 1+4=5

(b) Let $f : A \rightarrow B$, $g : B \rightarrow C$, $h : C \rightarrow D$ be three mappings. Prove that

(i) $h \circ (g \circ f) = (h \circ g) \circ f$

(ii) $f \circ i = f$ and $j \circ f = f$ where $i : A \rightarrow A$ and $j : B \rightarrow B$ are identity mappings.

(c) If the matrices A and B are commute, then show that A^{-1} and B^{-1} are also commute.

(d) Prove that every group of prime order is cyclic.

(e) Solve $x^4 - 2x^3 - 21x^2 + 22x + 40 = 0$ whose roots are in AP.

(f) Test the consistency and solve :

$$5x + 3y + 7z = 4$$
$$3x + 26y + 2z = 9$$
$$7x + 2y + 10z = 5$$

4. Answer ***any two*** : 5×2=10

(a) If α, β, γ are the roots of the equation $x^3 - px^2 + qx - r = 0$ then find the value of $\sum\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$ in terms of p, q and r.

(b) Find the condition that the cubic $x^3 - px^2 + qx - r = 0$ should have its roots in harmonic progression.

(c) If $f : A \to B$ and $g : B \to C$ be one-one and onto maps, then show that $g \circ f$ is inversible and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

5. Answer ***any two*** : 5×2=10

(a) Prove that the order of a cyclic group is equal to the order of any generator of the group.

(b) Prove that every finite group G is isomorphic to a permutation group.

(c) If $\cos^{-1}(\alpha + i\beta) = \theta + i\phi$, prove that

$\alpha^2 \operatorname{sech}^2\phi + \beta^2 \operatorname{cosech}^2\phi = 1.$

6. Answer **any two** : 5×2=10

(a) Prove that

$$(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\frac{\theta}{2}\cos\frac{n\theta}{2}$$

(b) Solve $x^3 - 3x - 1 = 0$ by Cardon's method.

(c) If H is a subgroup of G, then prove that there is a one to one correspondence between set of left coset of H in G and the set of right coset of H in G.

7. Answer **any two** : 5×2=10

(a) If $\tan(\theta + i\phi) = \cos\alpha + i\sin\alpha$, prove that

$$\theta = \frac{n\pi}{2} + \frac{\pi}{4} \text{ and } \phi = \frac{1}{2}\log_a \tan\left(\frac{\pi}{4} + \frac{\lambda}{2}\right)$$

(b) Prove that the necessary and sufficient condition for a matrix A to possess an inverse is that $|A| \neq 0$.

(c) Prove that every square matrix satisfies its own characteristic equation.
