Total number of printed pages-7

3 (Sem-1) MAT M 1 (f) Find the rank of the matrix

2021

(Held in 2022)

MATHEMATICS 2

(Major)

Examine whether the inverse of the

Paper: 1.1

(Algebra and Trigonometry)

Full Marks: 80

(h) Define stuod sorth: smiThe set of real

The figures in the margin indicate full marks for the questions.

- (i) What is normal form of a matrix? Answer the following questions: 1×10=10
- xo (a) Give an example of a relation on the set of real numbers R which is reflexive and transitive but not symmetric.
- Is generator of a cyclic group always 2. Give the answer of the foll? Supinu uence moduto ni is an equivalence
- ne (c) a Define Hermitian matrix.

3 (Sem-1) MAT M 1/0

Find all partitions of the set  $x = \{1, 2, 3\}$ .

- (e) Find the value of  $i^i$ .
- (f) Find the rank of the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{pmatrix}$$

- (g) Examine whether the inverse of the matrix  $\begin{pmatrix} 1 & w \\ w & w^2 \end{pmatrix}$  exists or not.
- (h) Define an operation \* on the set of real numbers R where  $a*b=a+2b, \forall a,b \in R$
- (i) What is normal form of a matrix?
- Find the amplitude of the complex number -1-i.
- 2. Give the answer of the following:  $2 \times 5 = 10$ 
  - (a) Can a non-Abelian group have an Abelian subgroup? Justify your answer.

(b) If  $f: A \to B$  and  $g: B \to C$  are bijective mappings, then prove that  $g \circ f$  is also a bijective mapping.

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(c) Prove that

ered w 
$$\pi = 2\sqrt{3} \left( 1 - \frac{1}{3 \cdot 3} + \frac{1}{5 \cdot 3^2} - \frac{1}{7 \cdot 3^3} + \dots \right)$$

by Find the spines of the cubic

- (d) Solve the equation  $x^3 + 6x + 20 = 0$  if one root is 1+3i.
- Show that the relation defined on  $N \times N$ (e) at reproby  $(a,b)\sim(c,d)$  iff a+d=b+c is an equivalence relation.
- $0 = 0 + x^2 + 21x^2 + 22x + 40 = 0$   $0 = 4 \times 6$  whose roots are in: **nuol grap** rawsan 3.
  - Define an equivalence relation on a (a) nonempty set. Show that the relation 'congruence modulo m' is an equivalence relation on the set of integers.

isomorphic to \$01448 the group.

(b) Let  $f: A \to B$ ,  $g: B \to C$ ,  $h: C \to D$  be three mappings. Prove that and a bijective mapping.

(i) 
$$h \circ (g \circ f) = (h \circ g) \circ f$$
  
(c) Prove that

(ii)  $f \circ i = f$  and  $j \circ f = f$  where  $i: A \rightarrow A$  and  $j: B \rightarrow B$  are identity mappings.

- (c) If the matrices A and B are commute, then show that  $A^{-1}$  and  $B^{-1}$  are also commute.
- (e) Show that the relation defined on N×N (d) Prove that every group of prime order is cyclic. equivalence relation.
- (e) Solve  $x^4 2x^3 21x^2 + 22x + 40 = 0$  whose roots are in AP. Which is work
- Define an equivalence relation on a Test the consistency and solve: congruence mast +3y +3y + 3y = onturgnoo z=0+1 .2153x+26y+2x0=1091101151=5 7x + 2y + 10z = 5

- 4. Answer any two: (11+11) 5×2=10
  - (a) If  $\alpha$ ,  $\beta$ ,  $\gamma$  are the roots of the equation  $x^3 px^2 + qx r = 0$  then find the value

of  $\sum \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$  in terms of p, q and r.

- $(b)^2$  Find the condition that the cubic  $x^3 px^2 + qx r = 0$  should have its shown roots in harmonic progression.
- (c) If  $f: A \to B$  and  $g: B \to C$  be one-one and onto maps, then show that  $g \circ f$  is expendence and onto maps, then show that  $g \circ f$  is done of the set of inversible and  $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ .
- 5. Answer any two:

5×2=10

(a) Prove that the order of a cyclic group is equal to the order of any generator of the group.

(b) Prove that every finite group G is isomorphic to a permutation group.

3 (Sem-1) MAT M 1/G

DIM TAM (Contd.) &

(c) If 
$$\cos^{-1}(\alpha + i\beta) = \theta + i\phi$$
, prove that not  $\alpha^2 \operatorname{sec} h^2 \phi + \beta^2 \operatorname{cosec} h^2 \phi = 1$ .

Solution and the prove that  $\alpha^2 \operatorname{sec} h^2 \phi + \beta^2 \operatorname{cosec} h^2 \phi = 1$ .

Solution is the prove that  $\alpha^2 \operatorname{sec} h^2 \phi + \beta^2 \operatorname{cosec} h^2 \phi = 1$ .

Answer any two:  $(\frac{\alpha}{\alpha} + \frac{\beta}{\alpha})$  in terms of p, q and r. (a) Prove that

$$(1+\cos\theta+i\sin\theta)^n + (1+\cos\theta-i\sin\theta)^n = 2^{n+1}\cos^n\frac{\theta}{2}\cos\frac{n\theta}{2}$$

$$(1+\cos\theta+i\sin\theta)^n + (1+\cos\theta+i\sin\theta)^n = 2^{n+1}\cos^n\frac{\theta}{2}\cos\frac{n\theta}{2}\cos\frac{n\theta}{2}$$

$$(1+\cos\theta+i\sin\theta)^n + (1+\cos\theta+i\sin\theta)^n = 2^{n+1}\cos^n\frac{\theta}{2}\cos\frac{n\theta}{2}\cos\frac{$$

- (b) Solve  $x^3 = 3x = 1 = 0$  by Cardon's method.
- (c) If H is a subgroup of G, then prove that there is a one to one correspondence between set of left coset of H in G and the set of right coset of H in G.

5x2=10

Answer any two:

Answer **any two**: (a) Prove that the order of a cyclic group is

 $5 \times 2 = 10$ 

to rots and (a) If  $tan(\theta + i\phi) = cos\alpha + i sin\alpha$ , prove that

$$\theta = \frac{n\pi}{2} + \frac{\pi}{4} \text{ and } \phi = \frac{1}{2} \log_a \tan\left(\frac{\pi}{4} + \frac{\lambda}{2}\right)$$
isomorphic to a permutation group.

- (b) Prove that the necessary and sufficient condition for a matrix A to possess an inverse is that  $|A| \neq 0$ .
- (c) Prove that every square matrix satisfies its own characteristic equation.