Total number of printed pages-7

where held, then

3 (Sem-3 /CBCS) MAT HC 1

2021

(Held in 2022)

MATHEMATICS

(Honours) aoiteant

Paper: MAT-HC-3016

(Theory of Real Functions)

Spelled to som Full Marks : 80 al 100 the

Time: Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed: $1 \times 10 = 10$

(a) Find
$$\lim_{x\to 2} \frac{x^3 = 40}{x^2 + 1}$$
 and let to too?

Between any two roots of the function

(b) Is the function $f(x) = x \sin\left(\frac{1}{x}\right)^{1}$ (i)

continuous at x=0? ded on $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$?

A Gen - A /CEOS | MAT JeC 1/O

Write the number of solutions of the

(c) Write the cluster points of A = (0,1).

.btno2 CBCS | MAT HC 1/G 2

```
Total number of printed pages-7
      f: (a, \infty) \to \mathbb{R} is such
                             (d) If a function
                      3 (Sem-3 /CBCS) MAT HC 1
          exist.
                       2021
                               \lim f(x) = 9
    (b) Let f (Held in 2022)
     ent lo Vii MATHEMATICS July
       function f(\operatorname{aruonoH}) + x^2, x \in \mathbb{R}.
              Paper: MAT-HC-3016
   Iser of Theory of Real Functions)
  S sels to sout Full Marks : 18021 1.1007 19
   si noisant: even function is
                              (a) The derivativ
  The figures in the margin indicate
        full marks for the questions.
 Between any two roots of the function
1.—Answer the following as directed: 1 \times 10 = 10
     root of the function \frac{x^3 + i4}{x^2 + 1} mid \lim_{x \to 2} \frac{x^3 + i4}{x^2 + 1}
                               be defined by
 If f(x) = |x^3| for x \in \mathbb{R}, then find
    (b) Is the function f(x) = x \sin\left(\frac{1}{x}\right)
          continuous at x=0? ded on [-1,1]?
 Write the number of solutions of the
     (c) Write the cluster points of A = (0,1).
3 (Sem - 3 /CBOS) MAT HC 1/0 : 3
                               3 (Sem-3 /CBCS) MAT HC 1/G
```

- (d) If a function $f:(a,\infty)\to\mathbb{R}$ is such that $\lim_{x\to\infty}xf(x)=L$, where $L\in\mathbb{R}$, then $\lim_{x\to\infty}f(x)=?$
 - (e) Write the points of continuity of the function $f(x) = \cos \sqrt{1 + x^2}$, $x \in \mathbb{R}$.
 - "Every polynomial of odd degree with real coefficients has at least one real roof." Is this statement true **or** false?
 - (g) The derivative of an even function is function. (Fill in the blank)
- (h) Between any two roots of the function $f(x) = \sin x, \text{ there is at least} ----$ root of the function $f(x) = \cos x$.

 (Fill in the blank)
 - (i) If $f(x) = |x^3|$ for $x \in \mathbb{R}$, then find f'(x) for $x \in \mathbb{R}$.

continuous at x=0 P

(j) Write the number of solutions of the equation ln(x) = x-2.

2. Answer the following questions: 2×5=10

- show that $\lim_{x\to 0} (x + sgn(x))$ does not exist. Show that $\lim_{x\to 0} (x + sgn(x))$ does not exist.
 - (b) Let f be defined for all $x \in \mathbb{R}$, $x \neq 3$ by that $f(x) = \frac{x^2 + x 12}{x + 3}$. Can f be defined at x = 3 in such a way that f is

? this gift is show that the function f(x) = |x| is

- (c) Show that $f(x) = x^2$ is uniformly and tacontinuous on [0, a], where a > 0. Show that $f(x) = x^2$ is uniformly on the substantial continuous of [0, a], where a > 0.
- a function is 'continuous at every point but whose derivative does not exist everywhere'.

 everywhere'.

 notice an example with justification that a function is 'continuous at every point does not exist everywhere'.

(e) Suppose $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 \sin \frac{1}{x^2}$, for $x \neq 0$ and

f(0) = 0. Is f(0) = 0 is f(0) = 0. In f(0) = 0. Is f(0) = 0. In f(0) = 0. It f(0)

3. Answer any four parts lol and 13 5×4=200

(a) If $A \subseteq \mathbb{R}$ and $f: A \to \mathbb{R}$ has a limit at $c \in \mathbb{R}$, then prove that f is bounded on some neighbourhood of c.

yd \mathcal{E}_{x} $x \neq 0$ His tol bennieb ed \mathcal{E}_{x} \mathcal{E}_{x}

 $\infty + = (x)f \times mil = (x)f \lim_{t \to \infty} f(x) = ($

(c) Show that the function f(x) = |x| is continuous at every point $c \in \mathbb{R}$.

(d) Give an example to show that the product of two uniformly continuous tand nonfunction is not uniformly continuous on R.

(e) Let $f:[a,b] \to \mathbb{R}$ be differentiable on [a,b]. If f' is positive on [a,b], then prove that f is strictly increasing on [a,b].

(f) Evaluatex—101 $\frac{1}{5x}$ siz 5x = (x)?

equation laft) o x

4. Answer any four parts: 10×4=40

(a) Let $f: A \to \mathbb{R}$ and let c be a cluster point of A. Prove that the following are equivalent.

(d) (i) Let I be $l = l(x) t^{\text{inilval}}(i)$ and let row is the self-up of the continuous on I.

For every sequence (x_n) in A that converges to c such that $x_n \neq c$ for

all $x \in \mathbb{N}$, the sequence $(f(x_n))$ converges to l.

 $f(x) = \frac{1}{1 + \sqrt{2}} \text{ for } x \in \mathbb{R} \text{ is uniformly}$

g such that f and g do not have limits at a point c but such that both f+g and fg have limits at c.

muminim-mumixem every but each (9) 6

of the and prove maximum.

(e) 6

(ii) Let $A \subseteq \mathbb{R}$, let $f: A \xrightarrow{\text{opposition}} \mathbb{R}^d$ and let c

be a cluster point of A. If $\lim_{x\to C} f(x)$

exists and if |f| denotes the exists and if |f| denotes the function defined for $x \in A$ by |f| and |f| for |f| functify. Proof that

 $\lim_{x\to c} |f|(x) = \left| \lim_{x\to c} f(x) \right|$

3 (Sem-3 /CBCS) MAT HC 1/G 5 0 D/I OH TAM (2080 Contd.) E

(c) Prove that the rational functions and
the sine functions are continuous on R. point of A. Prove that the following are countries.
(d) (i) Let I be an interval and let $f: I \to \mathbb{R}$ be continuous on I . Prove that the set $f(I)$ is an that interval. 5 The sequence of the set $f(I)$ is an $f(I)$ in
$f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$ $f(x) = \frac{1}{1+x^2} \text{ for } x \in \mathbb{R} \text{ is uniformly}$
limits at a point c but such that both f+g and fg have limits at c. muminim-mumixam avord bear state (a) 6 theorem. Let A \(\infty\) Let A \(\infty\) let f. The and let c
(x) f
$ f (x) = fx , \text{ Proof that}$ $\lim_{x \to c} f (x) = \lim_{x \to c} f(x)$

3 (Sem-3 /CBCS) MAT HC 1/G 6 7 D/1 DH TAM (2080) 8

(ii) If r is a rational number, let $f: \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{for } x \neq 0 \\ 0, & \text{otherwise} \end{cases}$$

Determine those values of r for which f'(0) exists.

- (g) State and prove Mean value theorem.

 Give the geometrical interpretation of the theorem. (2+5)+3=10
- (h) State and prove Taylor's theorem.

AND WATER TO THE TO

ro an a under while

mail that have the the pi to p

Harv agreement on the ac-

maring Degit Og. Ogs. w. D. of t