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3 (Sem-3/CBCS) MAT HC 3

2022

MATHEMATICS

(Honours)

Paper : MAT-HC-3036

(Analytical Geometry)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer **any ten** : $1 \times 10 = 10$

(i) Write down the formulae of transformation from one pair of rectangular axes to another with same origin.

(ii) Find the equation to the locus of the point $P(t, 2t)$ if t is a parameter.

Contd.

- (iii) For what value of a , the transformation $x' = -x + 2$, $y' = ax + 3$ is a translation?
- (iv) What is the locus represented by the equation $ax^2 - 5xy + 6y^2 = 0$?
- (v) Write down the polar equation of the straight line $x = 0$.
- (vi) Under what condition $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represent a pair of straight lines?
- (vii) What will be the equation of the line $ax + by + c = 0$ if the origin is transferred to the point (α, β) ?
- (viii) The parabola represented by the equation $y^2 = 4ax$ is not a closed curve. How can you justify it from the given equation?
- (ix) Write the relationship between the lengths of semi-major axis, semi-minor axis and the eccentricity for the standard equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $b > a$.

- (x) What conic does the following equation represent?

$$x^2 + 2xy + y^2 - 2x - 1 = 0$$

- (xi) What are the direction ratios of the normal to the plane given by equation $ax + by + cz + d = 0$?

- (xii) Write down the direction cosines of z-axis.

- (xiii) When does the equation $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ represent a sphere?

- (xiv) When is a plane said to be parallel to a line?

- (xv) Mention the condition under which the lines $\frac{x - \alpha_1}{l_1} = \frac{y - \beta_1}{m_1} = \frac{z - \gamma_1}{n_1}$ and

$$\frac{x - \alpha_2}{l_2} = \frac{y - \beta_2}{m_2} = \frac{z - \gamma_2}{n_2}$$
 are coplanar.

- (xvi) What are centre and radius of the sphere given by the equation

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0?$$

(xvii) Define the polar plane of a point (α, β, γ) with respect to the conicoid $ax^2 + by^2 + cz^2 = 1$:

(xviii) What are the coordinates of the vertex of the cone $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$?

2. Answer **any five** : $2 \times 5 = 10$

(a) Find the equation of the line $y = \sqrt{3}x$ when the axes are rotated through an angle $\frac{\pi}{3}$.

(b) If $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ are the extremities of any focal chord of the parabola $y^2 = 4ax$, prove that $t_1 t_2 = -1$.

(c) If the two pair of lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, prove that $pq + 1 = 0$.

(d) If e_1 and e_2 are the eccentricities of a hyperbola and its conjugate, show that $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$.

(e) Find the equation of the plane containing the lines $2x + 3y + 5z - 7 = 0$, $3x - 4y + z + 14 = 0$ and passing through the origin.

(f) Find the equation of the cone whose vertex is at the origin and whose guiding curve is given by $x = a, y^2 + z^2 = b^2$.

(g) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9, 2x + 3y + 4z = 5$ and the point $(1, 2, 3)$.

(h) Mention the conditions under which the general equation of the second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents (i) a parabola, (ii) an ellipse, (iii) a hyperbola, and (iv) a circle.

(i) Find the perpendicular distance of the point $(1, 4, -2)$ from the plane $2x - 3y + z = 5$.

- (j) The axis of a right circular cylinder is $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{2}$ and its radius is 5. Find its equation.

3. Answer **any four** : $5 \times 4 = 20$

- (a) Prove that the transformation of rectangular axes which converts

$$\frac{X^2}{P} + \frac{Y^2}{Q} \text{ into } ax^2 + 2hxy + by^2 \text{ will}$$

$$\text{convert } \frac{X^2}{P-\lambda} + \frac{Y^2}{Q-\lambda} \text{ into}$$

$$\frac{ax^2 + 2hxy + by^2 - \lambda(ab - h^2)(x^2 + y^2)}{1 - (a+b)\lambda + (ab - h^2)\lambda^2}$$

- (b) Prove that the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of parallel straight

$$\text{lines if } \frac{a}{h} = \frac{h}{b} = \frac{g}{f}.$$

- (c) Show that the line $lx + my = n$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if $a^2l^2 + b^2m^2 = n^2$.

- (d) Prove that the product of the perpendiculars from any point on a hyperbola to the asymptotes is constant.

- (e) A plane passes through a fixed point (p, q, r) , and cut the axes in A, B, C . Show that the locus of the centre of the sphere $OABC$ is $\frac{p}{x} + \frac{q}{y} + \frac{r}{z} = 2$.

- (f) Find the centre and radius of the circle $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$; $x - 2y + 2z = 3$.

- (g) The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the coordinate axes in A, B, C . Prove that the equation of the cone generated by the lines drawn from O is

$$yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0.$$

(h) Find the equation of the circular cylinder whose guiding circle is

$$x^2 + y^2 + z^2 = 9, x - y + z = 3.$$

4. Answer **any four** :

10×4=40

(a) If the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of straight lines, prove that the equation to the third pair of straight lines passing through the points, where these meet the axes is

$$ax^2 - 2hxy + by^2 + 2gx + 2fy + c + \frac{4fg}{c}xy = 0.$$

(b) Obtain the equation of the chord of the

conic $\frac{l}{r} = 1 + e \cos \theta$, joining the two

points on the conic, whose vectorial angles are $(\alpha + \beta)$ and $(\alpha - \beta)$.

(c) If PSP' and QSQ' are two perpendicular focal chords of a conic, prove that

$$\frac{1}{PS.SP'} + \frac{1}{QS.SQ'} = \text{a constant.}$$

(d) Find the equation of a polar of a given point $P(x_1, y_1)$ with respect to the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0.$$

Hence show that if the polar of a point P with respect to the conic passes through a point Q , then the polar of Q also passes through P .

(e) (i) Show that the locus of the points of intersection of perpendicular tangents to a parabola is its directrix.

(ii) Find the asymptotes of the hyperbola $xy + ax + by = 0$.

(f) What do you mean by skew lines? How do you define the shortest distance between two such lines? Find the length and the equations of the line of shortest distance between the lines

$$3x - 9y + 5z = 0, x + y - z = 0$$

$$6x + 8y + 3z - 13 = 0, x + 2y + z - 3 = 0.$$

(g) Prove that the radius of the circle in which the plane

$$\frac{x}{a} \sqrt{a^2 - b^2} + \frac{z}{c} \sqrt{b^2 - c^2} = \lambda \text{ cuts the}$$

$$\text{ellipsoid } \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \text{ is}$$

$$b \sqrt{1 - \frac{\lambda^2}{a^2 - c^2}}.$$

(h) Prove that the lines through (α, β, γ) at right angles to their polars with

$$\text{respect to } \frac{x^2}{a+b} + \frac{y^2}{2a} + \frac{z^2}{2b} = 1 \text{ generate}$$

the cone

$$(y - \beta)(\alpha z - \gamma x) + (z - \gamma)(\alpha y - \beta x) = 0.$$

What is the peculiarity of the case when $a = b$?

(i) Show that the equation of the cylinder whose generators are parallel to the line

$$\frac{x}{1} = \frac{y}{-2} = \frac{z}{3} \text{ and guiding curve is}$$

$$x^2 + 2y^2 = 1, z = 3 \text{ is}$$

$$3(x^2 + 2y^2 + z^2) + 8yz - 2zx + 6x - 24y - 18z + 24 = 0.$$

(j) What do you mean by a director sphere? Find the equation of the director sphere of the conicoid $ax^2 + by^2 + cz^2 = 1$. Hence or otherwise prove that the director sphere of the

$$\text{ellipsoid } \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \text{ is}$$

$$x^2 + y^2 + z^2 = a^2 + b^2 + c^2.$$