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3 (Sem–5/CBCS) MAT HE 4/5/6

2022

MATHEMATICS

(Honours Elective)

Answer the Questions from any one Option.

OPTION–A

Paper : MAT–HE–5046

(Linear Programming)

DSE(H)–2

Full Marks : 80

Time : Three hours

OPTION–B

Paper : MAT–HE–5056

(Spherical Trigonometry and Astronomy)

Full Marks : 80

Time : Three hours

OPTION–C

Paper : MAT–HE–5066

(Programming in C)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

Contd.

OPTION–A

Paper : MAT–HE–5046

(Linear Programming)

DSE (H)–2

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any ten*** questions from the following : *(Choose the correct answer)* 1×10=10

(i) A function $f(x)$ is said to the strictly convex at x if for two other distinct points x_1 and x_2

(a) $f\,[\lambda x_1 + (1-\lambda)x_2] < \lambda f\,(x_1) + (1-\lambda)\,f(x_2)$, where $0 \le \lambda \le 1$

(b) $f\,[\lambda x_1 + (1-\lambda)x_2] < \lambda f\,(x_1) + (1-\lambda)\,f(x_2)$, where $0 < \lambda < 1$

(c) $f\,[\lambda x_1 + (1-\lambda)x_2] \le \lambda f\,(x_1) + (1-\lambda)\,f(x_2)$, where $0 \le \lambda \le 1$

(d) None of the above

(ii) If X is the set of eight vertices of a cube, then the convex hull $C(X)$ is the

(a) surface of the cube

(b) vertices of the cubic

(c) whole cube

(d) None of the above

(iii) The extreme points of the convex set of feasible solutions are

(a) finife in number

(b) infinite in number

(c) either finite *or* infinite

(d) None of the above

(iv) In a linear programming problem

$$max\ Z = cx$$

subject to $Ax \geq b,\ x \geq 0$, c is called

(a) coefficient vector

(b) column vector

(c) price vector

(d) None of the above

(v) Consider a system $Ax = b$ of m equations in n unknowns, $n > m$. Then maximum number of basic solution is

(a) ${}^{m}C_{n}$

(b) ${}^{n}C_{m-1}$

(c) ${}^{n}C_{n-m}$

(d) None of the above

(vi) A basic feasible solution of an LPP is said to be non-degenerate BFS if

(a) none of the basic variable zero

(b) at least one of the basic variable zero

(c) exactly one of the basic variable zero

(d) None of the above

(vii) If the LPP $max\ Z = cx$ such that $Ax = b,\ x \geq 0$ has a feasible solution then at least one of the BFS will be

(a) maximal

(b) minimal

(c) optimal

(d) None of the above

(viii) If for any basic feasible solution of an LPP, there is some column α_j in A but not in B for $c_j - z_j > 0$ and $y_{ij} \leq 0\ (i = 1, 2, \cdots, m)$, then the problem has an unbounded solution if the objective function is to be

(a) maximized

(b) minimized

(c) *either* maximized *or* minimized

(d) None of the above

(ix) Standared form of LPP is

(a) Min $Z = cx$ s.t. $Ax \geq b,\ x \geq 0$

(b) Max $Z = cx$ s.t. $Ax \leq b,\ x \geq 0$

(c) Max $Z = cx$ s.t. $Ax \geq b,\ x \geq 0$

(d) None of the above

(x) The incoming vector in a simplex table will be taken as α_k if

(a) $\Delta_k = max\ \Delta_j$

(b) $\Delta_j = max\ \Delta_k$

(c) entries of α_k are all negative

(d) None of the above

(xi) If we consider dual of an LPP, then in the dual the requirement vector of the primal problem becomes

(a) objective function

(b) price vector

(c) variable

(d) None of the above

(xii) The necessary and sufficient condition for any LPP and its dual to have optimal solution is that

(a) both have basic solution

(b) both have unbounded solution

(c) both have feasible solution

(d) None of the above

(xiii) If any of the constraint in the primal is perfect equality, the corresponding dual variable is

(a) perfect equality

(b) unrestricted in sign

(c) strictly inequality

(d) None of the above

(*xiv*) In an assignment problem

$$\min Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} ,$$

$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \cdots, n$ means that

(*a*) only one job is done by i-th person $i = 1, 2, \cdots, n$

(*b*) i-th person is assigned to j-th job

(*c*) only one person should be assigned to the j-th job, $j = 1, 2, \cdots, n$

(*d*) None of the above

(*xv*) In a transportation problem if we apply North-West Corner method, we always get

(*a*) non-degenerated BFS

(*b*) degenerated BFS

(*c*) optimal solution

(*d*) None of the above

(xvi) For optimality test in a transportation problem, number of allocation in independent position must be.

(a) $m+n$

(b) $m+n+1$

(c) $m+n-1$

(d) None of the above

(xvii) In a transportation table for cell evaluation we use the formula

(a) $c_{rs}=u_r+v_s$, $d_{ij}=(u_i+v_j)-c_{ij}$

(b) $c_{rs}=u_r+v_s$, $d_{ij}=c_{ij}-(u_i+v_j)$

(c) $c_{rs}=u_r+v_s$, $d_{ij}=u_i+v_j$

(d) None of the above

(xviii) Define finite game a "Game Theory".

2. Answer ***any five*** from the following : 2×5=10

(a) Show that the FS $x_1=1$, $x_2=0$, $x_3=1$ and $z=6$ to the system of equations

$x_1+x_2+x_3=2$

$x_1-x_2+x_3=2$, $x_j\geq 0$ which minimize

$z=2x_1+3x_2+4x_3$ is not basic.

(b) Is $x_1 = 1,\ x_2 = \frac{1}{2},\ x_3 = x_4 = x_5 = 0$ a basic solution to the following system?

$$x_1 + 2x_2 + x_3 + x_4 = 2$$

$$x_1 + 2x_2 + \frac{1}{2}x_3 + x_5 = 2$$

(c) Examine convexity of the set

$$S = \left\{(x_1, x_2) : x_1^2 + x_2^2 \leq 4\right\}$$

(d) Define artificial variable. Give an example.

(e) Define unbounded solution of an LPP. How can we determine that the solution of an LPP is unbounded?

(f) What is two phase method to solve an LPP? Mention the phases?

(g) Write "complementary slackness theorem" of a dual problem.

(h) How can we find entering vector in a simplex table?

(i) Write the 'Test of optimality' for primal dual method.

(j) What is cost matrix of an assignment problem?

3. Answer ***any four*** from the following : 5×4=20

(a) Prove that the set of all feasible solutions of an LPP is a convex set. [Assume that the set is non empty]

(b) If the objective function of an LPP assume its optimal value at more than one extreme point, then prove that every convex combination of these extreme points gives the optimal value of the objective function.

(c) Give the dual of the following LPP :

$$Max\ Z = 2x_1 + 3x_2 + x_3$$

$$\text{s.t.}\quad 4x_1 + 3x_2 + x_3 = 6$$

$$x_1 + 2x_2 + 5x_3 = 4$$

$$x_1, x_2, x_3 \geq 0$$

(d) Solve the following transportation problem by North-West Corner method

	S_1	S_2	S_3	S_4	a_i
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
b_j	20	40	30	40	100

(e) Solve the game whose pay-off matrix is

$$\begin{bmatrix} -1 & -2 & 8 \\ 7 & 5 & -1 \\ 6 & 0 & 12 \end{bmatrix}$$

(f) Mark the feasible region represented by the constraint conditions

$x_1 + x_2 \le 1$, $3x_1 + x_2 \ge 3$, $x_1 \ge 0$, $x_2 \ge 0$

(g) Find initial BFS of the following LPP :

$Max\ Z = 2x_1 + 3x_2$

s.t. $-x_1 + 2x_2 \le 4$

$x_1 + x_2 \le 6$

$x_1 + 3x_2 \le 9$

x_1, x_2 unrestricted

(h) If in an assignment problem, a constant is added or substracted to every element of row (or column) of the cost matrix $[c_{ij}]$, then prove that an assignment which minimizes the total cost for one matrix, also minimize the total cost for the other matrix.

4. Answer ***any four*** from the following:

10×4=40

(a) A soft drink plant has two bottling machines A and B. It produces and sells 8 ounce and 16 ounce bottles. The following data is available

Machine	8 ounce	16 ounce
A	100/minute	40/minute
B	60/minute	75/minute

The machines can run 8 hours per day, 5 days per week. Weekly production of drink cannot exceed 3,00,000 ounces and the market can absorb 25,000 eight ounce bottles and 7,000 sixteen ounce bottles per week, Profit on these bottle is 15 paise and 25 paise per bottle respectively. The planner wishes to minimize his profit subject to all the production and marketing restrictions. Formulate it as an LPP and solve graphically.

(b) State and prove fundamental theorem of LPP.

(c) Using simplex alogorithm solve the problem

$$Max\ Z = 2x_1 + 5x_2 + 7x_3$$

$$\text{s.t. } 3x_1 + 2x_2 + 4x_3 \le 100$$
$$x_1 + 4x_2 + 2x_2 \le 100$$
$$x_1 + x_2 + 3x_3 \le 100$$
$$x_1, x_2, x_3 \ge 0$$

(d) Use dual to solve the LPP

$$Min\ Z = 2x_1 + x_2$$

$$\text{s.t. } 3x_1 + x_2 \ge 3$$
$$4x_1 + 3x_2 \ge 6$$
$$x_1 + 2x_2 \ge 3$$
$$x_1, x_2 \ge 0$$

(e) Solve the following transportation problem :

Market

Plant	A	B	C	D	Available
X	10	22	10	20	8
Y	15	20	12	8	13
Z	20	12	10	15	11
Required	5	11	8	8	32

(f) State and prove Fundamental Duality theorem.

(g) The pay-off matrix for A in a two persons zero sum game is given below. Determine the value of the game and the optimum strategies for both players

		B: I	II	III
	I	−1	2	1
A	II	1	−2	2
	III	3	4	−3

(h) Find an optimal solution of the following LPP without using simplex method :

$$Max\ Z = 2x_1 + 3x_2 + 4x_3 + 7x_4$$

$$\text{s.t.} \quad 2x_1 + 3x_2 - x_3 + 4x_4 = 8$$

$$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$$

$$x_i \geq 0, \ i = 1, 2, 3, 4$$

(i) If an LPP

$$Max\ Z = cx \ \text{ s.t. } \ Ax = b, \ x \geq 0$$

where A is $mX(m+n)$ matrix of co-efficients given by $A = (\alpha_1, \alpha_2, \cdots, \alpha_{m+n})$, has at least one feasible solution, then prove that it has at least one basic feasible solution.

(j) A company has four territories open and four salesman available for assignment. The territories are not equally rich in their sales potential; it is estimated that a typical salesman operating in each territory would bring in the following annual sales

Territories	I	II	III	IV
Annual sales (Rs.)	60,000	50,000	40,000	30,000

The four salesman are also considered to differ in ability; it is estimated that, working under the same conditions, their yearly sales would be proportionally as follows :

Salesman :	A	B	C	D
Proportion :	7	5	5	4

If the criterion is maximum expected total sales, the intuitive answer is to assign the best salesman to the richest territory, the next best salesman to the second richest and so on. Verify this answer by the assignment technique.

OPTION-B

Paper : MAT–HE–5056

(*Spherical Trigonometry and Astronomy*)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any ten*** of the following questions : $1 \times 10 = 10$

(i) Define spherical triangle.

(ii) Define great circle and small circle.

(iii) How many great circles can be drawn through two given points, when the points are the extremities of a diameter?

(iv) What are the relations between the elements of a spherical triangle and that of its polar triangle?

(v) Define hour angle of a heavenly body.

(vi) What is the point on the celestial sphere whose latitude, longitude, right ascension and declination, all are zero?

(vii) Name the *two* points in which the elliptic cuts the equator on the celestial sphere.

(viii) What is the declination of the pole of the ecliptic ?

(ix) What is parallatic ellipse ?

(x) What do you mean by circumpolar star ?

(xi) State the third law of Kepler.

(xii) Define polar triangle.

(xiii) What is the duration of a day and night at equinoxes ?

(xiv) Explain, what is meant by rising and setting of stars ?

(xv) Where does the celestial equator cut the horizon ?

(xvi) Define right ascension of a heavenly body.

(xvii) What are the altitude and hour angle of the zenith ?

(xviii) State the cosine formula related to a spherical triangle.

2. Answer ***any five*** of the following questions : $2\times5=10$

(a) Draw a neat diagram of the celestial sphere showing the horizontal coordinates of a heavenly body.

(b) Prove that the sides and angles of a polar triangle are respectively the supplements of the angles and sides of the primitive triangle.

(c) State Newton's law of gravitation.

(d) ABC is an equilateral spherical triangle, show that $\sec A = 1 + \sec a$.

(e) Give the usual three methods for locating the position of a star in space.

(f) Prove that the altitude of the celestial pole at any place is equal to the latitude of the place of the observer.

(g) Discuss the effect of refraction on sunrise.

(h) Drawing a neat diagram, discuss how horizontal coordinates of a heavenly body are measured.

(i) Prove that the altitude of the celestial pole at any place is equal to the latitude of that place.

(j) Show that right ascension α and declination δ of the sun is always connected by the equation
$\tan\delta = \tan\varepsilon \sin\alpha$, ε being obliquity of the ecliptic.

3. Answer **_any four_** questions of the following : 5×4=20

(a) In a spherical triangle *ABC*, prove that
$\cos a = \cos b \cos c + \sin b \sin c \cos A$

(b) In a spherical triangle *ABC*, if $b + c = \pi$, then prove that
$\sin 2B + \sin 2C = 0$.

(c) At a place in north latitude ϕ, two stars *A* and *B* of declinations δ and δ_1 respectively, rise at the same moment and *A* transists when *B* sets. Prove that
$\tan\phi \tan\delta = 1 - 2\tan^2\phi \tan^2\delta_1$

(d) If ψ is the angle which a star makes at rising with the horizon, prove that $\cos\psi = \sin\phi \sec\delta$, where the symbols have their usual meanings.

(e) Deduce Kepler's laws from the Newton's law of gravitation.

(f) Write short notes on : (i) Zodiac (ii) Morning star and Evening star.

(g) If v_1, v_2 be the velocities of two planets in their orbits, and T_1, T_2, be the respective distances from the sun, prove that $v_2 : v_1 = \sqrt{T_1} : \sqrt{T_2}$.

(h) Prove that the altitude of a star is the greatest when it is on the meridian of the observer.

4. Answer ***any four*** questions of the following : 10×4=40

(a) In a spherical triangle ABC, prove that

$$\frac{\sin a}{\sin A} = \sqrt{\frac{1 - \cos a \cos b \cos c}{1 + \cos A \cos B \cos C}}$$

(b) If ψ is the angle at the centre of the sun subtended by the line joining two planets at distance a and b from the sun at stationary points, show that

$$\cos\psi = \frac{\sqrt{ab}}{a + b - \sqrt{ab}}$$

(c) If the inferior ecliptic limits are $\pm\varepsilon$ and if the satellite revolves n times as fast as the sun, and its node regrades θ every revolution the satellite makes round its primary, prove that there cannot be fewer consecutive solar eclipses at one node then the integer next less than $\dfrac{2(n-1)\varepsilon}{n\theta+2\pi}$.

(d) What is Cassin's hypothesis? Under this hypothesis, show that the amount of refraction R can be found from $\tan\phi = \dfrac{\sin R}{\mu - \cos R}$, where μ is the refractive index of the atmosphere with respect to vacuum and ϕ is the angle of refraction at certain point on the upper surface of the atmosphere.

(e) Explain the effects of refraction on right ascension and declination.

(f) State Kepler's laws of planetary motion. If V_1 and V_2 are the linear velocities of a planet at perihelion and aphelion respectively and e is the eccentricity of the planets orbit, prove that

$$(1-e)V_1 = (1+e)V_2.$$

(g) Define astronomical refraction and state the laws of refraction. Derive the formula for refraction as $R = k\tan\xi$. ξ being the apparent zenith distance of a heavenly body. Mention *one* limitation of this formula.

(h) A star of declination δ is seen on the prime vertical. Show that its declination is increased by $\frac{2K\left(\sin^2\phi - \sin^2\delta\right)}{\sin 2\delta}$ due to refraction, ϕ being the latitude of the place.

(i) Assuming the planetary orbits to be circular and coplanar, prove that the sidereal period P and the synodic period S of an inferior planet are related to the earth's periodic time E by

$$\frac{1}{S} = \frac{1}{P} - \frac{1}{E}$$

Calculate the sidereal period (in mean solar days) of a planet whose sidereal period is same as its synodic period.

(j) Prove that, if the fourth and higher powers of e are neglected,

$$E = M + \frac{e \sin M}{1 - e \cos M} - \frac{1}{2}\left(\frac{e \sin M}{1 - e \cos M}\right)^3$$

is a solution of Kepler's equation in the form.

OPTION–C

Paper : MAT–HE–5066

(Programming in C)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any seven*** questions : 1×7=7

(a) Write the output of a :

int a;

a=5/2;

(b) Write one arithmetic and one logical operator in C.

(c) What is a global variable ?

(d) Name the header file that is used to compile the function '*sqrt* (x)'.

(e) Which of the following can be used as a variable : x1, x_1, x%1 ?

(f) Write *two* reserved words used in C language.

(g) Convert the following mathematical expression into a C expression :

$$z = \frac{5x+6}{3x^2+2} - \frac{\sin x^2}{\sqrt{x}}$$

(h) State whether True **or** False :
C-language is case-sensitive.

(i) Write *any two* built-in functions used in C-language.

(j) For $x = 5$, $y = 2$, write the output of $x \% y$.

(k) Write the utility of getch ( ) function.

(l) Define a *two*-dimensional array.

2. Answer ***any four*** questions : 2×4=8

(a) What is the difference between C character and C string ?

(b) Write *four* different C statements each adding 1 to integer variable x.

(c) Name *any four* functions available in 'stdio.h'.

(d) Write a C program that will input a character and give output, the same.

(e) int a, b, temp;

$a = 5$;

$b = 3$;

temp = a;

$a = b$;

b = temp;

Write the output of 'a' and 'b'.

(f) Write the general syntax of *scanf*( ) function to read the integer variable a.

(g) Write the syntax of 'nested if' statement in C language.

(h) Write the output of the following:

$c = 0$

for $(i = 1; i \leq 5; i++)$

$c = c + i;$

3. Answer ***any three*** parts : 5×3=15

(a) Write a C program to calculate the commission for a sales representative as per the sales amount given below:

if sales ≤ 500, commission is 5% of sales

if sales > 500 but ≤ 2000, commission is Rs. 35 plus 10% above Rs. 500 of sales

if sales > 2000 but ≤ 5000, commission is Rs. 185 plus 12% above Rs. 2000 of sales

if sales > 5000, commission is 12.5% of sales

(b) Write a C program to find the average of best three marks from the given four test marks.

(c) Give a general syntax of 'switch' statement in C.

Write the outputs of a and b of the following :

(i)	(ii)	(iii)
$a=5;$	$a=5;$	$a=5;$
$b=7;$	$b=7;$	$b=7;$
$if\ (a>b)$	$if\ (a>b)$	$\text{if}\ (a>b \,\|\, a<b)$
$\{a=a+1;$	$a=a+1;$	$a=a+1;$
$b=b+1\};$	$b=b+1$	

(d) Write a C program to print integers from 1 to n omitting those integers which are divisible by 7.

(e) Write a C program to generate the Fibonacci series up to n terms.

(f) Write a C program to find the sum of squares of all integers between 1 and n.

(g) Write a C program to print the $n \times n$ zero marix.

(h) Write a C program to add 1 to each element of a 3×3 matrix.

4. Answer ***any three*** parts : 10×3=30

(a) Write the differences between 'while loop' and 'do-while' loop using examples. Write a C program to check whether the given number is an Armstrong number. (An Armstrong number is one that is equal to the sum of cubes of individual digits. For *ex.* $153 = 1^3 + 5^3 + 3^3$) 5+5=10

(b) Develop a C program to compute the value of π from the series

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \cdots$$

Write a C program to convert a binary number into a decimal number. 5+5=10

(c) Write a C program for each of the following : 5+5=10

(i) to find the mean and standard deviation of any n values.

(ii) to add two matrices of order $m \times n$.

(d) Write a C program to compute the value of e^x using the series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

For this build two functions—one to find the factorial and the other to compute x^n, for a given n.

(e) Write a C program to find the LCM of two numbers a and b, where b is the sum of the digits of a. Use two functions—one is to find LCM and the other is to find the sum of the digits.

($gcd.lcm = a.b$) 10

(f) Write the syntax of 'nested for' loop and show with a suitable C program. What are the differences between 'break' statement and '*exit*( )' function. Write a C program using 'break' statement, and write the outputs. Also write the outputs of the same program if the 'break' statement is replaced by '*exit*( )' function. 1+4+2+3=10

(g) What is meant by recursive function? What is its use? Demonstrate the use of recursive function by a suitable C program. 2+2+6=10

(h) What are the uses of 'continue' and 'goto' statements in a C program? Explain each with a suitable C program segment. 5+5=10
