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3 (Sem–5/CBCS) PHY HE 3

2024

PHYSICS

(Honours Elective)

Paper : PHY–HE–5036

(Advanced Mathematical Physics–I)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 7 = 7$

(a) What do you mean by basis of a vector space ?

(b) How can be obtained an orthonormal set of vectors from an orthogonal set ?

(c) What is called an abelian group ?

(d) Find $\ln A$, where $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$.

(e) What is Einstein's summation

Contd.

convention ?

(f) Show that $\delta_{ij}\varepsilon_{ijk} = 0$.

(g) If A_i and A^i represent first rank covariant and contravariant tensors respectively, prove that $A_i = A^i$ in Cartesian coordinate system.

2. Answer the following questions : 2×4=8

(a) Determine whether the vectors (1, 2, 3) and (2,–2,0) are linearly independent or not.

(b) State and verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. 1+1=2

(c) Using tensor notations, prove that $\vec{\nabla} \times (\vec{\nabla}\phi) = 0$

(d) What is Minkowski space ? What are the transformation equations relating coordinates in this space ? 1+1=2

3. Answer ***any three*** questions from the following : 5×3=15

(a) (i) Difine a group by meationing axioms. 2½

(ii) Show that the set of $n \times n$ unitary matrices forms a group under matrix multiplication. 2½

(b) (i) Using tensor notations, show that $\operatorname{div} \vec{A}$ is an invariant. 3

(ii) Prove that diagonalizing matrix of a real symmetrie matrix is orthogonal. 2

(c) (i) Using direction cosines, establish the relation $\overline{x}_i = a_{ij} x_j$.

(ii) Write the inverse transformation equation.

4+1=5

(d) (i) State quotient law of tensors. 2

(ii) Prove that the sum of two tensors of the same rank and type is also a tensor of same rank and type.

3

(e) (i) Show that the properly of symmetry of a tensor between a pair of dissimilar indices is not invariant under coordinate transformation. 3

(ii) If A^{ij} is an anti-symmetric tensor and B_i is a vector, show that

$$A^{ij}B_iB_j = 0.$$ 2

4. Answer the following question : *(a)* **or** *(b)*, *(c)* **or** *(d)* *and* *(e)* **or** *(f)* 10×3=30

(a) (i) Show that the set of all complex numbers form a vector space over the field of real numbers. 4

(ii) What is the dimension of above mentioned vector space? Justify your answer. 1+2=3

(iii) Show that $A^k = ED^kE^{-1}$, where k is any integer, D and E are diagonal and diagonalizing matrices of matrix A. 3

Or

(b) (i) Evaluate e^A, where $A = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$ 7

(ii) Find the standard matrix of linear transformation T from R^2 to R^4 such that

$T(e_1) = (3,1,3,1), T(e_2) = (-5,2,0,0),$

where $e_1 = (1,0)$ and $e_2 = (0,1)$. 3

(c) (i) Solve the following coupled differential equations:

$$\frac{dx}{dt} = x + y \text{ and}$$

$$\frac{dy}{dt} = 4x + y$$

using method of matrices where $x(0) = y(0) = 1$. 7

(ii) Find the anti-symmetric tenser of rank two associated with the vector $(x, x+y, x+y+z)$ in three-dimensional space. 3

Contd.

Or

(d) (i) Establish the relation

$dS^2 = g_{ij}\, dx^i\, dx^j$, where symbols have their usual meanings. 4

(ii) Show that

$\varepsilon_{iks}\, \varepsilon_{mps} = \delta_{im}\, \delta_{kp} - \delta_{ip}\delta_{km} = 0$. 3

(iii) If $A^{\lambda}\, B_{\mu\nu}$ is a tensor for all first rank contravariant tensors A^{λ} then show that $B_{\mu\nu}$ is also a tensor. 3

(e) (i) Using tensor analysis, prove the following vector identities :

$\vec{\nabla}.\left(\vec{A}\times\vec{B}\right) = \vec{B}.\left(\vec{\nabla}\times\vec{A}\right) - \vec{A}.\left(\vec{\nabla}\times\vec{B}\right)$ and

$\vec{A}\times\left(\vec{B}\times\vec{C}\right) = \vec{B}.\left(\vec{A}\times\vec{C}\right) - \vec{C}.\left(\vec{A}.\vec{B}\right)$

2+3=5

(ii) Using tensor analysis, establish the relation $L_i = \varepsilon_{ijk} I_{jk}$ where symbals have their usual meanings. 3

(iii) If the length of a vector is invariant under coordinate transformation (rotation), show that $a_{ij}a_{il} = \delta_{jl}$. 2

Or

(i) Derive with seat diagrams, the components of stress at a point of a solid body in three-dimensional space. 5

(ii) Use tensor analysis to find the components of a vector in plane polar coordinates whose components in Cartesian coordinates are $\dot{x}$ and $\dot{y}$. 5