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1 (Sem-4) PHY 4

2025

PHYSICS

Paper : PHY0400404

(Mathematical Physics)

Full Marks : 45

Time : Two hours

The figures in the margin indicate full marks for the questions.

1. Answer to the following questions :

1×5=5

- (a) Express the Laplace equation for two-dimensional Cartesian system.
- (b) What is the necessary condition for the existence of the Fourier's series ?

- (c) If certain complex function $f(z)$ is analytic then which equation should it satisfy ?
- (d) What is the residue of a function at a simple pole ?
- (e) What is the fundamental property of Levi-Civita symbol, ϵ_{ijk} ?

2. Answer **any five** questions : $2 \times 5 = 10$

- (a) Express the general form of Laplace equation in spherical polar coordinate system.
- (b) State the Dirichlet conditions for the existence of a Fourier series.
- (c) State the Cauchy-Riemann conditions in Cartesian coordinates.
- (d) Express the difference between a pole and a branch point with examples.

- (e) Define a symmetric tensor with an example.
- (f) What is the significance of Einstein's summation convention ?
- (g) Write down the probability mass function of the Poisson distribution and illustrate the parameters within.
- (h) Find the mean and variance of a binomial distribution with parameters n and p .
- (i) Express the Fourier series of a function $f(x) = x$ defined within the bound $(-\pi, \pi)$.
- (j) What type of boundary conditions are used to solve the wave equation for a vibrating string fixed at both ends ?

3. Answer **any four** questions : $5 \times 4 = 20$

- (a) Solve the one-dimensional wave equation for a string of length L fixed at both ends using the separation of variables method (Give the general form of the solution and mention the boundary conditions).
- (b) Solve the Laplace's equation in two dimensions for a rectangular region with suitable boundary conditions using separation of variables method (express the solution graphically without evaluating the arbitrary constants).
- (c) Find the Fourier series (sine and cosine form) for the function $f(x) = x^2$ in the interval $(-\pi, \pi)$. Show all steps clearly.
- (d) Expand the periodic square wave function in a complex Fourier series and write down the expression.

(e) State and prove Cauchy's integral formula for a function analytic in a simply connected domain. (State all assumptions clearly)

(f) Determine the nature and the order of the singularity of the function,

$$f(z) = \frac{\sin z}{z^3} \text{ at } z = 0, \text{ and compute its}$$

residue.

(g) If $A^{\mu\nu}$ and $B_{\mu\nu}$ are two tensors of rank two, then describe their transformation rules under coordinate transformation and also show that $A^{\mu\nu}B_{\mu\nu}$ is an invariant quantity.

(h) Starting from the Poisson distribution, derive the condition under which it approximates the binomial distribution. Explain the physical or statistical significance of this limit.

4. Answer **any one** question : $10 \times 1 = 10$

(a) Solve the following differential equation using separation of variable method.

$$\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u, \text{ where } u(x, 0) = 6e^{-3x}.$$

$$(b) \text{ Given, } f(x) = \begin{cases} -1, & \text{for } -\pi < x < -\frac{\pi}{2}, \\ 0, & \text{for } -\frac{\pi}{2} < x < \frac{\pi}{2}, \\ +1, & \text{for } \frac{\pi}{2} < x < \pi \end{cases}$$

find the Fourier expression of $f(x)$.

(c) Evaluate the integral $\oint_C \frac{e^z}{z^2(z-1)} dz$,

where C is the positively oriented circle $|z| = 2$, using residue theorem. Clearly identify the singularities of the integrand, determine the order of each singularity, compute the residues at the poles enclosed by C . $6+4=10$

(d) Prove the quotient law of tensors. Show that the Kronecker delta δ_j^i behaves as a mixed tensor. What is its rank? Using tensor transformation laws, show that it acts as the identity operator under index contraction. $4+3+1+2=10$